

Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 1 (Individual)

香港数学竞赛 (1995 – 96)

决赛项目 1 (个人)

- (i) The perimeter of an equilateral triangle is exactly the same in length as the perimeter of a regular hexagon. The ratio of the areas of the triangle and the hexagon is  $2 : a$ , find the value of  $a$ .

$a =$

若一个等边三角形与一个正六边形周长相等，而其面积的比为  $2 : a$ ，求  $a$  的值。

- (ii) If  $5^x + 5^{-x} = a$  and  $5^{3x} + 5^{-3x} = b$ , find the value of  $b$ .

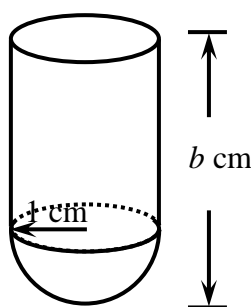
$b =$

若  $5^x + 5^{-x} = a$  和  $5^{3x} + 5^{-3x} = b$ ，求  $b$  的值。

- (iii) The figure shows an open cylindrical tube (radius = 1 cm) with a hemispherical bottom of radius 1 cm. The height of the tube is  $b$  cm and the external surface area of the tube is  $c\pi \text{ cm}^2$ . Find the value of  $c$ .

$c =$

图中为一圆柱体和半球体组成的无盖空心物体。半球体和圆柱体的半径均为 1 cm。若这物体的长度为  $b$  cm，且表面面积为  $c\pi \text{ cm}^2$ ，求  $c$  的值。



- (iv) Two fair dice are thrown. Let  $d$  be the probability of getting the sum of scores to be  $\frac{c}{6}$ . Find the value of  $d$ .

$d =$

抛掷两粒正常骰子，设取得点数总和是  $\frac{c}{6}$  的概率为  $d$ ，求  $d$  的值。

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Final Event 2 (Individual)

香港数学竞赛 (1995 – 96)

决赛项目 2 (个人)

- (i) It is given that  $m, n > 0$  and  $m + n = 1$ . If the minimum value of

$\left(1 + \frac{1}{m}\right)\left(1 + \frac{1}{n}\right)$  is  $a$ , find the value of  $a$ .

$a =$

已知  $m, n > 0$  和  $m + n = 1$ 。若  $\left(1 + \frac{1}{m}\right)\left(1 + \frac{1}{n}\right)$  的最小值为  $a$ ，求  $a$  的值。

- (ii) If the roots of the equation  $x^2 - (10 + a)x + 25 = 0$  are the square of the roots of the equation  $x^2 + bx = 5$ , find the positive value of  $b$ .

$b =$

方程  $x^2 - (10 + a)x + 25 = 0$  的根是  $x^2 + bx = 5$  的根的平方。求  $b$  的正数值。

- (iii) If  $(xy - 2)^{b-1} + (x - 2y)^{b-1} = 0$  and  $c = x^2 + y^2 - 1$ , find the value of  $c$ .

$c =$

若  $(xy - 2)^{b-1} + (x - 2y)^{b-1} = 0$  及  $c = x^2 + y^2 - 1$ ，求  $c$  的值。

- (iv) If  $f(x)$  is a polynomial of degree two,  $f(f(x)) = x^4 - 2x^2$  and  $d = f(c)$ , find the value of  $d$ .

$d =$

若  $f(x)$  是一二次多项式， $f(f(x)) = x^4 - 2x^2$  及  $d = f(c)$ ，求  $d$  的值。

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Final Event 3 (Individual)

香港数学竞赛 (1995 – 96)

决赛项目 3 (个人)

- (i) If  $a$  is a real number and  $2a^3 + a^2 - 275 = 0$ , find the value of  $a$ .

$a =$

若  $a$  为实数及  $2a^3 + a^2 - 275 = 0$ , 求  $a$  的值。

- (ii) Find the value of  $b$  if  $3^2 \cdot 3^5 \cdot 3^8 \cdots 3^{3b-1} = 27^a$ .

$b =$

若  $3^2 \cdot 3^5 \cdot 3^8 \cdots 3^{3b-1} = 27^a$ , 求  $b$  的值。

- (iii) Find the value of  $c$  if  $\log_b(b^c - 8) = 2 - c$ .

$c =$

若  $\log_b(b^c - 8) = 2 - c$ , 求  $c$  的值。

- (iv) If  $\left[(4^c)^c\right]^c = 2^d$ , find the value of  $d$ .

$d =$

若  $\left[(4^c)^c\right]^c = 2^d$ , 求  $d$  的值。

Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 4 (Individual)

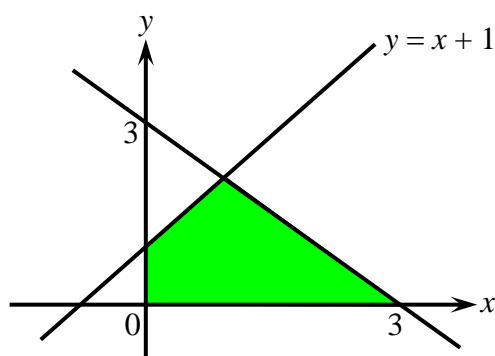
香港数学竞赛 (1995 – 96)

决赛项目 4 (个人)

- (i) In the figure, the area of the shaded region is  $a$ . Find the value of  $a$ .

图中阴影部分面积是  $a$ 。求  $a$  的值。

$a =$



- (ii) If  $8^b = 4^a - 4^3$ , find the value of  $b$ .

$b =$

若  $8^b = 4^a - 4^3$ ，求  $b$  的值。

- (iii) Given that  $c$  is the positive root of the equation  $x^2 - 100b + \frac{10000}{x^2} = 0$ , find the value of  $c$ .

$c =$

已知  $c$  是方程式  $x^2 - 100b + \frac{10000}{x^2} = 0$  之正根，求  $c$  的值。

- (iv) If  $d = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{(c-1) \times c}$ , find the value of  $d$ .

$d =$

若  $d = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{(c-1) \times c}$ ，求  $d$  的值。

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Final Event 5 (Individual)

香港数学竞赛 (1995 – 96)

决赛项目 5 (个人)

- (i) Four fair dice are thrown. Let  $a$  be the probability of getting at least half of the outcome of the dice to be even. Find the value of  $a$ .

$a =$

同时投掷四颗骰子。设取得最小一半骰子的结果为偶数的概率为  $a$ ，求  $a$  的值。

- (ii) It is given that  $f(x) = \frac{3}{8}x^2(81)^{-\frac{1}{x}}$  and  $g(x) = 4\log_{10}(14x) - 2\log_{10} 49$ . Find the value of  $b = f\{g[16(1-a)]\}$ .

$b =$

已知  $f(x) = \frac{3}{8}x^2(81)^{-\frac{1}{x}}$  和  $g(x) = 4\log_{10}(14x) - 2\log_{10} 49$ ，求  $b = f\{g[16(1-a)]\}$  的值。

- (iii) Let  $c = \frac{1}{b^2-1} + \frac{1}{(2b)^2-1} + \frac{1}{(3b)^2-1} + \cdots + \frac{1}{(10b)^2-1}$ , find the value of  $c$ .

$c =$

设  $c = \frac{1}{b^2-1} + \frac{1}{(2b)^2-1} + \frac{1}{(3b)^2-1} + \cdots + \frac{1}{(10b)^2-1}$ ，求  $c$  的值。

Hint 提示：  $\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x+1}\right)$

- (iv) In the following diagram,  $PC$  is a tangent to the circle (centre  $O$ ) at the point  $P$ , and  $\triangle ABO$  is an isosceles triangle,  $AB = OB$ ,  $\angle PCO = c$  and  $d = \angle QPC$ , where  $c, d$  are radian measures. Find the value of  $d$ . (take  $\pi = 22/7$ )

$d =$

在右图中， $PC$  是圆（圆心为  $O$ ）的切线，切点在  $P$ 。 $\triangle ABO$  是等腰三角形， $AB = OB$ ， $\angle PCO = c$  及  $d = \angle QPC$ ，其中  $c, d$  为弧度。求  $d$  的值。（取  $\pi = 22/7$ ）

